Cyber-physical systems and Lyapunov functions

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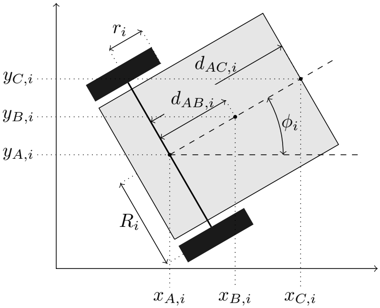
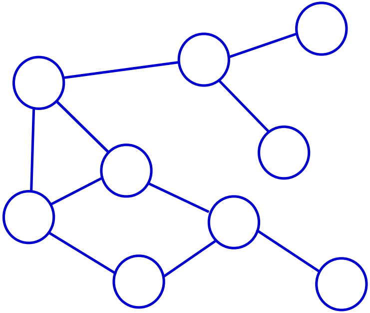
Outline

- Cyber-physical Lyapunov functions
- Energy functions for microgrids
- Control under DoS

Cyber-physical Lyapunov functions

A second-order networked system



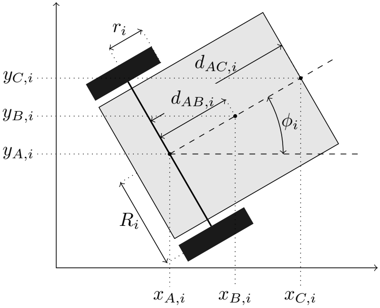


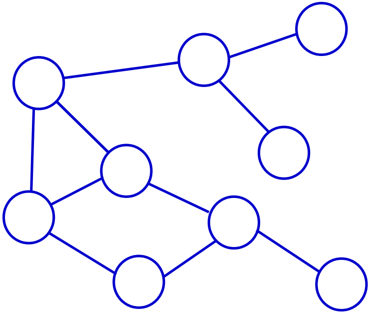
A second-order networked system

$$\begin{aligned} \dot{p}_i &= v_i \\ \dot{v}_i &= -v_i + u_i, \quad i \in \mathcal{I} = \{1, 2, \ldots, n\} \end{aligned}$$

$p_i \in \mathbb{R}^p$ position
$v_i \in \mathbb{R}^p$ velocity
$u_i \in \mathbb{R}^p$ torque

A second-order networked system





Rendezvous

For each initial condition $p_i(0), z_i(0)$, $i = 1, 2, \ldots, n$,

$$\begin{aligned} \lim_{t\to+\infty} \|p_i(t) - p_j(t)\| &= 0, \quad \forall i, j \\ \lim_{t\to+\infty} \|v_i(t)\| &= 0, \quad \forall i \end{aligned}$$

Formation control

Virtual coupling

System i is interconnected to its neighbors via

$$u_i = \sum_{j \in \mathcal{N}_i} \psi_{ij}(z_{ij}),$$

with $\psi_{ij} : \mathbb{R} \to \mathbb{R}$, $\mathcal{C}^1$, nondecreasing and odd and

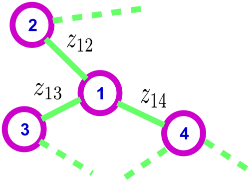
$$z_{ij} = p_j - p_i$$

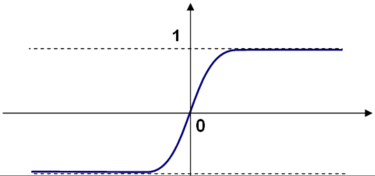
Overall

In compact form

$$\begin{aligned} \dot{p} &= v \\ \dot{v} &= -v - D\Psi(D^T p) \\ &= -v - D\Psi(z) \end{aligned}$$

D incidence matrix of the graph
$\Psi = col(\psi_1 \ldots, \psi_m)$





Energy-based analysis

Networked system + virtual coupling

$$\begin{aligned} \dot{p}_1 &= v_1 & \dot{p}_2 &= v_2 \\ \dot{v}_1 &= -v_1 + \psi(p_2 - p_1) & \dot{v}_2 &= -v_2 + \psi(p_1 - p_2) \end{aligned}$$

Energy-based (Lyapunov) analysis I

Consider 2 agents ($n = 2$) evolving on a line ($p_i, v_i \in \mathbb{R}$) and let

$$q = (z, v), \quad \text{with} \quad z = p_2 - p_1$$

and define

$$\begin{aligned} U_{\text{phys}}(q) &:= \underbrace{\frac{1}{2}(v_1^2 + v_2^2)}_{\textit{kinetic}} + \underbrace{\int_0^z \psi(s)ds}_{\textit{potential}} \\ &= \tfrac{1}{2} v^T v + \int_0^z \mathbb{1}^T \Psi(s)ds \end{aligned}$$

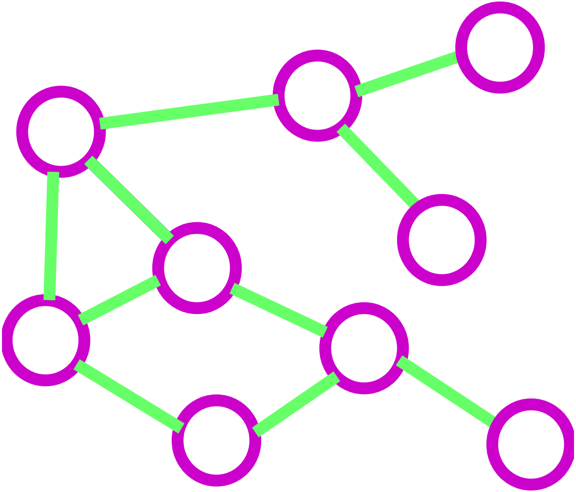
Energy-based (Lyapunov) analysis II

$$\begin{aligned}\frac{d}{dt}U_{\text{phys}}(q) = \quad &= -v_1^2 - v_2^2 + (v_1 - v_2)\psi(z) + \psi(z)(v_2 - v_1) \\ &= -v_1^2 - v_2^2\end{aligned}$$

- Energy is dissipated until system comes to a stop
- If $v = 0$ and $z \neq 0$ then virtual force $\psi(z)$ kicks in
- The system comes to a stop iff $z = 0$

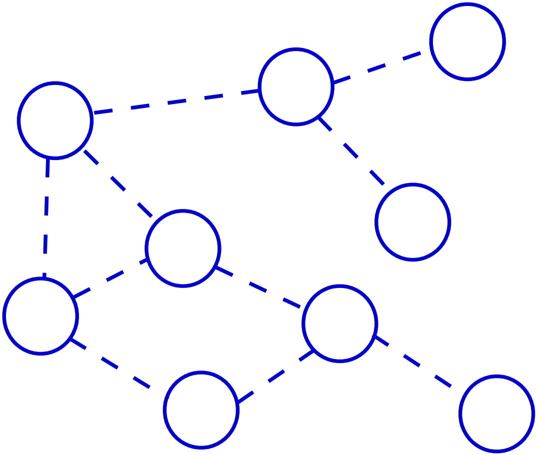
Ideal scenario

- Continuous measurements
- Continuous control updates



Cyber-physical scenario

- To limit network usage
- To reduce sensor wear
- To reduce actuator wear



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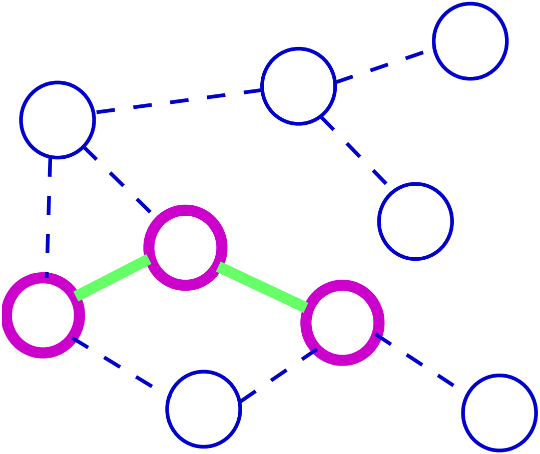

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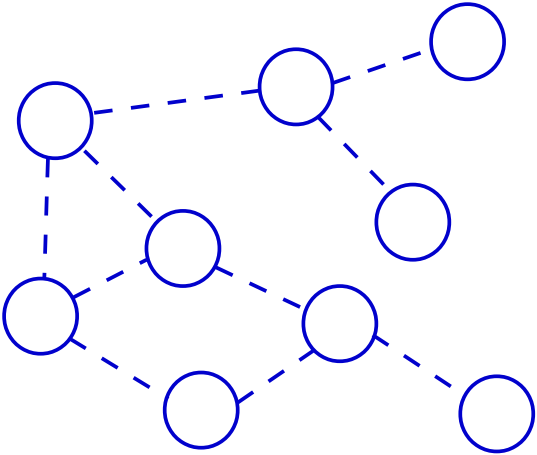
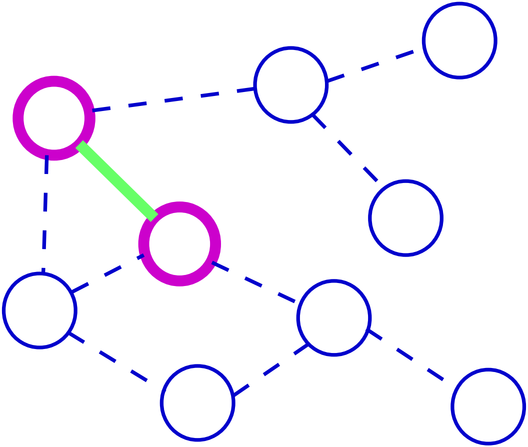


Problem statement

- To limit network usage
- To reduce sensor wear
- To reduce actuator wear



Problem statement

Communication/computation limitations Agents update their control and/or take their measurements at t_ℓ^{ij}, $\ell \in \mathbb{Z}$,

$$u_i = \sum_{j \in \mathcal{N}_i} \psi_{ij}(\hat{z}_{ij})$$

where

$$\begin{cases} \dot{\hat{z}}_{ij}(t) = 0, & t \neq t_\ell^{ij} \\ \hat{z}_{ij}(t^+) = z_{ij}, & t = t_\ell^{ij} \end{cases}$$

Problem

For each agent i and each neighbor $j \in \mathcal{N}_i$, determine sequence t_ℓ^{ij} so that rendezvous is achieved.

Energy-based analysis

Energy function $n = 2$

$$U_{\text{phys}}(q) := \underbrace{\frac{1}{2}(v_1^2 + v_2^2)}_{\textit{kinetic}} + \underbrace{\int_0^z \psi(s)ds}_{\textit{potential}}$$

Energy-based (Lyapunov) analysis

$$\begin{aligned}\frac{d}{dt}U_{\text{phys}}(q) = \quad & = \quad -v_1^2 - v_2^2 + (v_1 - v_2)\boldsymbol{\psi(\hat{z})} + \boldsymbol{\psi(z)}(v_2 - v_1) \\ & \neq \quad -v_1^2 - v_2^2\end{aligned}$$

Due to the sampling, energy may **not** be dissipated

Cyber-physical (Lyapunov) energy function

$$U(q) \quad := \quad U_{\text{phys}}(q) + \boldsymbol{U_{\text{cyber}}(q)}$$

where

$$U_{\text{phys}}(q) := \frac{1}{2}(v_1^2 + v_2^2) + \int_0^z \psi(s)ds$$

and

$$\boldsymbol{U_{\text{cyber}}(q) := \frac{1}{2}\phi \cdot \left(\psi(\hat{z}) - \psi(z)\right)^2}$$

is the “energy” of the sampling error **weighted** via **positive** ϕ .

Analysis

Lyapunov analysis

$$
\begin{aligned}
\frac{d}{dt}U(q) &= -v_1^2 - v_2^2 + (v_1 - v_2)\boldsymbol{\psi(\hat{z})} + \boldsymbol{\psi(z)}(v_2 - v_1) \\
&\quad -\frac{1}{2}\boldsymbol{\frac{d\phi}{dt}}\big(\psi(\hat{z}) - \psi(z)\big)^2 \\
&\quad -\phi\big(\psi(\hat{z}) - \psi(z)\big)\nabla\psi(z)(v_2 - v_1)
\end{aligned}
$$

The choice of $\boldsymbol{\frac{d\phi}{dt}}$ as

$$
\frac{d\phi}{dt} = -\frac{1}{\sigma}(1 + \phi^2 (\nabla\psi(z))^2)
$$

and a completion of the squares argument yields

$$
\frac{d}{dt}U(q) \le (-1 + 2\sigma)(v_1^2 + v_2^2) \le 0.
$$

where σ measures the convergence degradation.

Clock

The "weight" $\phi \in [a, b]$ plays the role of a clock

Clock dynamics

$$\begin{array}{rcll} \dot{\phi} & = & -\frac{1}{\sigma}\left(1 + \phi^2 \left(\nabla\psi(z)\right)^2\right) & \phi \in [a, b], \\ \phi^+ & = & b & \phi = a. \end{array}$$

where

- $\sigma \in (0, \frac{1}{2})$
- $0 < a < b$

Event-triggered control requires z
Self- and time-triggered implementations avoid this

Event-based coordination

Theorem

1. The set

$$\{(p, v, \hat{z}, \phi) \,:\, p_1 = p_2 = \ldots = p_n,\, v = \mathbf{0},\, \hat{z} = 0 \text{ and } \phi \in [a, b]^n\}$$

   is **globally asymptotically stable**
2. The solutions have a **uniform semiglobal dwell-time**

- Second-order heterogeneous **nonlinear** agents in $\mathbb{R}^p$

$$\begin{aligned} \dot{p}_i &= y_i \\ \dot{v}_i &= f_i(v_i, u_i) \\ y_i &= h_i(v_i) \end{aligned}$$

- General coordination problems $\boldsymbol{z}_{ij} \to \boldsymbol{\mathcal{A}}_{ij}$

- **Self-** and **time-triggered** rules

De Persis–Postoyan. *A Lyapunov redesign of coordination algorithms for cyberphysical systems*. **IEEE Transactions on Automatic Control** `arXiv 1404.0576`

Lyapunov functions and microgrids

A microgrid model

A network reduced micro-grid model

$$\begin{aligned} \dot{\theta} &= \omega \\ T_P \dot{\omega} &= -(\omega - \omega^*) - K_P(P - P^*) + u_P \\ T_Q \dot{V} &= f(V, Q, u_Q) \end{aligned}$$

$\theta \in \mathbb{T}^n$ voltage phase angles
$\omega \in \mathbb{R}^n$ frequency
$V \in \mathbb{R}^n$ voltage magnitudes

Active and reactive power

$$\begin{aligned} P_i &= \sum_{j \in \mathcal{N}_i} B_{ij} V_i V_j \sin \theta_{ij}, \quad \theta_{ij} := \theta_i - \theta_j \\ Q_i &= B_{ii} V_i^2 - \sum_{j \in \mathcal{N}_i} B_{ij} V_i V_j \cos \theta_{ij}, \quad \theta_{ij} := \theta_i - \theta_j \end{aligned}$$

Comparison $\underbrace{T_P \dot{\omega}}_{M\dot{v}} = \underbrace{-(\omega - \omega^*)}_{-(v - v^*)} - \underbrace{K_P(D\Gamma(V)\mathbf{sin}(D^T\theta) - P^*)}_{D\Psi(D^T p) - D\Psi(D^T p^*)} + u_P$

Voltage dynamics

$f(V, Q, u_Q)$ represents various voltage dynamics/controllers

	$\boldsymbol{f(V, Q, u_Q)}$	$\boldsymbol{u_Q}$
Droop	$-V - K_Q Q + u_Q$	$-V^* - K_Q Q^*$
Quadratic droop	$-[V]V - K_Q Q + [V]u_Q$	V^*
Reactive current	$-[V]^{-1}Q + u_Q$	$[\overline{V}]^{-1}\overline{Q}$
Reactive consensus	$-[V]K_Q L_Q K_Q Q + [V]u_Q$	$K_Q L_Q K_Q \overline{Q}$

Droop Zhong-Hornik '12, Schiffer *et al* '14
Quadratic droop Simpson-Porco *et al* '15
Reactive current Machowski *et al* '08, DP-Monshizadeh '15
Reactive consensus Schiffer *et al* '15

$$T_{Q,i}\dot{V}_i = V_i K_{Q,i} \sum_{j=1}^{n} a_{ij}^{Q}(K_{Q,j}Q_j - K_{Q,i}Q_i) + V_i u_{Q_i}$$

Energy functions

$$U_{\text{phys}}(q) \;:=\; \underbrace{\frac{1}{2}\omega^T K_P^{-1} T_P \omega}_{kinetic} + \underbrace{\int_0^z \mathbb{1}^T \Gamma(V) \mathbf{sin}(D^T s) ds}_{potential} + H(V)$$

	$H(V)$
Droop	$-\mathbb{1}^T K_Q V - (\overline{Q} + K_Q^{-1}\overline{V})\boldsymbol{ln}(V)$
Quadratic droop	$\frac{1}{2} V^T K_Q^{-1} V$
Reactive current	0
Reactive consensus	$-\overline{Q}^T \boldsymbol{ln}(V)$

Lyapunov (energy) functions are crucial to

- Obtain large signal stability analysis (**no linearization**)
- **Remove** frequency-voltage **decoupling** assumption
- Interconnect with **dynamic feedback**

Frequency synchronization and voltage stability

Reactive power consensus Solutions to microgrid dynamics

$$
\begin{aligned}
\dot{\theta} &= \omega \\
T_P\dot{\omega} &= -(\omega-\omega^*) - K_P(P-P^*) + u_P \\
T_Q\dot{V} &= -[V]K_Q L_Q K_Q Q + [V]u_Q
\end{aligned}
$$

in closed-loop with $u_Q = \overline{u}_Q$ and

$$
\begin{aligned}
\dot{\xi} &= -L_P\xi + K_P^{-1}(\omega^*-\omega) \\
u_P &= \xi
\end{aligned}
$$

locally converge to $\omega = \omega^*$ and $V = V^*$, where

$$
\mathbb{1}^T K_Q^{-1}\,\mathbf{ln}(V(t)) = \mathbb{1}^T K_Q^{-1}\,\mathbf{ln}(V^*) = \mathbb{1}^T K_Q^{-1}\,\mathbf{ln}(V(0))
$$

Power sharing

Active power sharing If droop coefficients are selected proportionally $(k_P)_i P_i^* = (k_P)_j P_j^*$, then

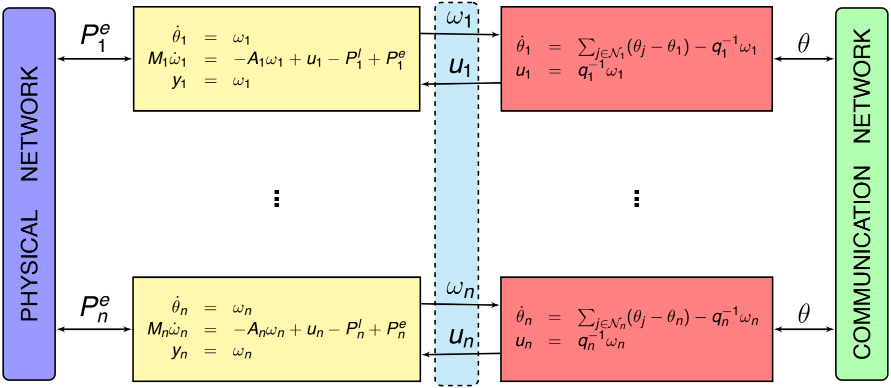
$$(k_P)_i \overline{P}_i = (k_P)_j \overline{P}_j \quad \forall i, j$$

Reactive power consensus

$$(k_Q)_i \overline{Q}_i = (k_Q)_j \overline{Q}_j$$

De Persis–Monshizadeh. *A modular design of incremental Lyapunov functions for microgrid control with power sharing*. `arXiv 1404.0576`

Power networks as a cyber-physical system



Control under Denial of Service

Cyber-physical systems

Data loss due to human action

- Number of documented cyber attacks have increased very rapidly in recent years
  - S. Amin, A. Cárdenas, and S. Sastry, 2009
  - Y. Mo, T. Hyun-Jin Kim, K. Brancik, D. Dickinson, H. Lee, A. Perrig, and B. Sinopoli, 2012
- Cyber attacks in the form of Denial-of Service (DoS) can be trivially launched against wireless-based communication infrastructures
  - K. Pelechrinis, M. Iliofotou and S. Krishnamurthy, 2011

This part of the talk

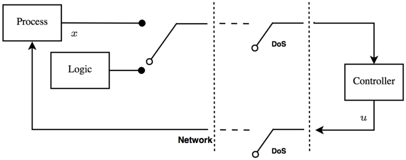
Stabilization of linear control systems under DoS attacks on the feedback channel

Framework

Process

$$\dot{x}(t) = Ax(t) + Bu(t) + w(t)$$

Block diagram



Logic Device responsible for generating the sampling sequence

$$\{t_k\}, \quad k \in \mathbb{N}, \quad t_0 := 0$$

Controller Sample-and-hold

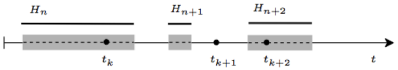
$$u_{\text{ideal}}(t) = K\,x(t_k), \quad \forall\, t \in [t_k, t_{k+1}[$$

Denial-of-Service (DoS)

Denial of Service

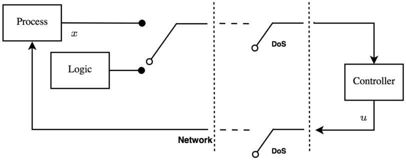
$$H_n := [h_n, h_n + \tau_n[, \quad n \in \mathbb{N}, \quad h_0 \geq 0$$

H_n n-th DoS interval
τ_n duration of H_n



Actual control
$u(t) = Kx(t_{k(t)})$, $\quad t_{k(t)}$ time of the last successful update

Problem formulation



Stabilization in the presence of DoS

Design the logic generating the sampling sequence $\{t_k\}$ such that the closed-loop system

$$\dot{x}(t) = Ax(t) + BKx(t_{k(t)}) + w(t)$$

is **input-to-state stable**, namely

$$\|x(t)\| \leq \alpha e^{-\beta t} \|x(0)\| + \gamma \|w_t\|_\infty$$

Admissible DoS signals

Admissible DoS

1. The DoS sequence $\{h_n\}$, $n \in \mathbb{N}$, is such that

$$\inf_{n \in \mathbb{N}} \tau_n = \tau_* > 0$$

2. there exist constants $\kappa \in \mathbb{R}_{\geq 0}$ and $p \in (0, 1)$ such that

$$|\Xi(t)| \leq \kappa + pt$$

for all $t \in \mathbb{R}_{\geq 0}$

where

$$\Xi(t) := \left(\bigcup_{n \in \mathbb{N}} H_n \right) \bigcap [0, t]$$

is the total interval of DoS within $[0, t]$

Sampling logic

Control gain K is such that $A + BK$ is Hurwitz

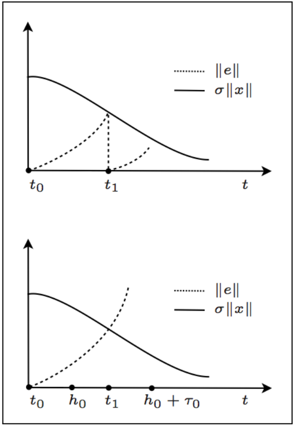
$$\dot{x}(t) = Ax(t) + BKx(t_{k(t)}) = (A + BK)x(t) + BKe(t)$$

Control update law Define the sampling error

$$e(t) := x(t_{k(t)}) - x(t)$$

and the control update law [Tabuada 2007]

$$\|e(t)\| \leq \sigma\|x(t)\|, \quad \forall\, t \notin \Xi(t)$$



Main result

Closed-loop system ($\boldsymbol{w}(t) = 0$)

$$\Sigma \; : \; \dot{x}(t) = Ax(t) + BKx(t_{k(t)}) = (A + BK)x(t) + BKe(t)$$

Theorem

There exist

$$\gamma_1(A, B, K, \sigma), \gamma_2(A, B, K, \sigma) > 0$$

such that Σ is GES for any DoS sequence satisfying

$$|\Xi(t)| \; \leq \; \kappa + pt$$

with

$$p \; < \; \frac{\omega_1}{\omega_1 + \omega_2}$$

Key inequality $V(x(t)) \leq \mathrm{e}^{-\omega_1(t - |\Xi(t)|) + \omega_2 |\Xi(t)|)} V(x(0))$

Comments

Lyapunov analysis of cyberphysical systems under DoS permits extension to

- **nonlinear systems**[1]
- **network systems**[2]

De Persis-Tesi **Input-to-State Stabilizing Control under Denial-of-Service.** *IEEE Transactions on Automatic Control*, 1–15, 10.1109/TAC.2015.2416924.

[1] De Persis–Tesi. **On resilient control of nonlinear systems under Denial-of-Service**. *Proc. 53rd IEEE-CDC*, 5254 - 5259, 2014.

[2] Senejohnny–Tesi–De Persis. **Self-triggered coordination over a shared network under Denial-of-Service**. *Proc. 54th IEEE-CDC*, 2015.

Conclusions

Conclusions

- Lyapunov (energy) functions for complex networks
- Cyber-physical Lyapunov function
- Robustness to sampling and data loss

	CPL	**MC**	**DoS**
CPL		$\star$	$\star$
MC	$\star$		+CPL $\star$
DoS	$\star$	+CPL $\star$	

QUESTIONS?